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Simultaneous Substitution and Utility Maximization: An MRS Analysis with a Fixed Budget

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ABSTRACT

This paper models consumer choice, allowing simultaneous buying and selling of two goods. Unlike standard models, consumers endogenously determine their trading position. A fixed budget component represents savings or borrowing. Using a Cobb-Douglas utility function, we show optimal consumption requires equating the Marginal Rate of Substitution (MRS) with an effective price ratio, incorporating buy/sell decisions. Deviations from this condition drive simultaneous substitution, increasing utility. The model highlights how consumers actively reshape their consumption bundles through trade, offering a more realistic perspective than traditional approaches.

Keywords: Simultaneous Substitution, Endogenous Determination, Cobb-Douglas Utility Function, Marginal Rate of Substitution, Pre-committed Expenditure, Effective Price

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1. Introduction

Traditional consumer choice models typically assume that individuals allocate a given income across a set of goods, taking prices as exogenous and purchasing non-negative quantities of each good (Mas-Colell et al., 1995; Varian, 2014). This paper relaxes the assumption of unidirectional transactions (buying only) and introduces the concept of simultaneous substitution. We develop a model where consumers can simultaneously be buyers of one good and sellers of another, effectively trading goods to achieve a more preferred consumption bundle. This endogenous trading behavior is a crucial, yet often overlooked, aspect of real-world consumer decision-making. Examples include trading goods within households (Becker, 1981; Chiappori, 1988), participating in barter economies, reselling purchased items (e.g., on online marketplaces), or even exchanging services.

The model incorporates a fixed budget component, denoted by A, which adds further realism. This component can represent savings and deposits ($A>0$), borrowing or pre-committed expenditures ($A<0$), or a standard budget constraint (when $A=0$). The inclusion of A allows us to analyze how existing financial positions influence the consumer's trading and consumption decisions, connecting to the literature on intertemporal choice (Deaton, 1992; Browning 1996).

The paper's central focus is the Marginal Rate of Substitution (MRS). We demonstrate that the consumer optimizes by equating their MRS – their subjective valuation of the trade-off between goods – to an *effective* price ratio. This effective price ratio is not simply the ratio of market prices; it incorporates the consumer's decision to be a buyer or seller of each good, represented by parameters α and β. When the MRS deviates from this effective price ratio, the consumer has a clear incentive to engage in simultaneous substitution, selling some of the good they value less and buying more of the good they value more, thereby reaching a higher indifference curve and increasing their overall utility. This dynamic is reminiscent of, but distinct from, the adjustments visualized in an Edgeworth box framework (Edgeworth, 1881).

2. Literature Review

This paper contributes to several related areas within microeconomic theory:

2.1. Standard Consumer Theory

The model builds upon the foundational principles of utility maximization subject to a budget constraint (Varian, 2014; Nicholson, 2012; Pindyck, 2018). However, it departs significantly from the standard model by endogenizing the direction of trade for each good.

2.2. General Equilibrium with Trade

General equilibrium models often incorporate trading, typically at the market level, focusing on the existence and properties of equilibrium prices (Arrow & Debreu, 1954; McKenzie, 1954). This paper, in contrast, provides a detailed microeconomic model of *individual* trading behavior as a means of utility maximization. While general equilibrium considers interactions between *many* agents, our model focuses on the optimal decision of a *single* agent who can both buy and sell.

2.3. Household Economics

The model has relevance to the study of intra-household resource allocation, where family members may exchange goods or services (Becker, 1981; Chiappori, 1988). The fixed budget component (A) can be interpreted as the household's overall resource constraint.

2.4. Intertemporal Choice

The inclusion of the fixed budget component (A) allows for a connection to intertemporal consumption and saving decisions (Deaton, 1992; Browning 1996). A positive A can be seen as a form of saving, while a negative A represents borrowing.

2.5. Behavioral Economics

A negative value for A could also be interpreted through the lens of behavioral economics, relating to concepts such as mental accounting and pre-commitment devices (Thaler, 1985; Liabson, 1997). Consumers might pre-commit to certain expenditures, reducing their available budget.

2.6. Edgeworth Box

The concept of optimal allocation where MRSs are equalized is a central concept in the Edgeworth Box framework (Edgeworth, 1981). Our model can be seen as a generalization where the "other trader" is the market.

2.7. Transaction Costs in Consumer Choice

Although our model doesn't explicitly incorporate transaction costs, it's relevant to the broader literature that considers how costs associated with trading affect consumer behavior (Heller, 1978; Kruz, 1974).

This paper's key contribution is its explicit modeling of *simultaneous substitution* at the *individual consumer* level. It moves beyond the standard assumption of unidirectional purchases and provides a framework for analyzing how consumers actively reshape their consumption bundles through trading to achieve higher utility.

3. Presentation of the SUM Model and Contemporary Consumer Behavior

3.1. Contemporary Consumer Behavior and the Capgemini Report

Contemporary consumer behavior has become a central focus of recent research. A significant contribution is the Capgemini Research Institute's report What Matters to Today's Consumers: 2024 Consumer Behavior Tracker for the Consumer Product and Retail Industries (Bridet & Mazza, 2024). It examines shifting consumer expectations in the retail and product sectors.

The report identifies five broad themes influencing consumer behavior. These include the impact of generative artificial intelligence on purchasing decisions, the rapid growth of social commerce, and heightened consumer concern with sustainability and product origin. Importantly, consumer choice today reflects not only functional needs such as price and quality but also ethical considerations, signaling a shift toward multidimensional decision-making. The SUM model can be regarded as an extension to not only EKB model, but also to digital marketing which is growing rapidly.

3.2. Consumer Behavior in the Era of Digitalization

Digitalization has made understanding consumer behavior paramount for marketing and business growth. As Sahu and Gita (2024) emphasize, consumers in digital environments are more aware, better informed, and increasingly active participants in markets.

Through e-marketing, consumers now both buy and sell across platforms, blurring traditional roles. Digital tools also enable personalization at scale, fostering closer relationships between brands and consumers. This dual role of consumer—as both informed buyer and occasional seller—represents one of the most important features of contemporary behavior in the digital era.

3.3. The Engel–Kollat–Blackwell (EKB) Model in Contemporary Contexts

The Engel–Kollat–Blackwell (EKB) model remains one of the most widely applied frameworks for studying consumer decision-making. Recent studies have adapted the model to new contexts, particularly digital marketplaces. A 2025 McKinsey report highlights five forces shaping consumer behavior: the influence of local brands, the impact of social media, greater attention to sustainability, reliance on peer reviews, and the growing use of AI in consumer journeys (McKinsey, 2025).

These findings confirm that the EKB framework continues to provide valuable insights into dynamic consumer environments while accommodating modern behavioral drivers.

3.4. Applications of the EKB Model

Empirical studies illustrate the flexibility of the EKB model. Putri and Widyastuti (2022) applied it to online shopping in Indonesia, showing that trust and authenticity matter alongside convenience and price. Sari and Wijaya (2023) used the model to analyze decision-making in the automotive industry, finding that consumers weigh both functional product attributes and symbolic factors such as brand reputation.

Such research demonstrates the model's adaptability to sector-specific contexts while reinforcing its relevance in explaining diverse purchasing behaviors.

3.5. The Evolving Consumer Landscape

Industry reports also stress the broader evolution of consumer behavior. A 2023 McKinsey study underscores that preferences are increasingly fluid, shaped by global uncertainty, inflation, supply disruptions, and cultural change. Businesses, therefore, must adopt adaptive strategies that anticipate emerging needs rather than simply reacting to established trends (McKinsey, 2023).

4. The Model

Goods: Two goods, G_1 and G_2, with prices P_1 and P_2, respectively.

4.1. Budget Constraint:

$$TM = A + \alpha G_1 P_1 + \beta G_2 P_2$$

TM : Total Money available (exogenous). This represents the consumer's total purchasing power

A: Fixed Budget Component (exogenous). This represents resources or obligations *independent* of the current consumption choice:

$A > 0$: Savings, a reserve fund, or previously acquired wealth

$A < 0$: Borrowing, a pre-committed expenditure (e.g., a loan payment), or a debt

$A = 0$: The standard budget constraint, where all available money is allocated to G_1 and G_2

α : Trading parameter for G_1:

$\alpha = 1$: Consumer is a net *buyer* of G_1, $\alpha = -1$: Consumer is a net *seller* of G_1, $\alpha = 0$: Consumer does not trade G_1

β : Trading parameter for G_2:

$\beta = 1$: Consumer is a net *buyer* of G_2, $\beta = -1$: Consumer is a net *seller* of G_2, $\beta = 0$: Consumer does not trade G_2

The combination of α and β allows for four distinct trading scenarios: buying both goods $(\alpha = 1, \beta = 1)$, selling both goods $(\alpha = -1, \beta = -1)$, buying G_1 and selling G_2 $(\alpha = 1, \beta = -1)$, and selling G_1 and buying G_2 $(\alpha = -1, \beta = 1)$.

4.2. Utility Function (Cobb-Douglas)

$$TU = U(G_1, G_2) = CG_1^a G_2^b$$

C : A positive constant representing the overall level of utility (scaling factor).

a : The output elasticity of G_1 $(0 < a < 1)$. This represents the relative importance of G_1 in the consumer's utility.

b : The output elasticity of G_2 $(0 < b < 1)$. This represents the relative importance of G2 in the consumer's utility.

4.3. Optimization Problem

The consumer's goal is to maximize their total utility (TU) subject to the budget constraint.

Lagrangian: $L = CG_1^a G_2^b + \lambda(TM - A - \alpha G_1 P_1 - \beta G_2 P_2)$ where λ is the Lagrange multiplier, representing the marginal utility of income.

4.3.1. First-Order Conditions (FOCs)

$$\frac{\partial L}{\partial G_1} = aCG_1^{a-1} G_2^b - \lambda \alpha P_1 = 0 \rightarrow \frac{aU(G_1, G_2)}{G_1} = \lambda \alpha P_1$$

$$\frac{\partial L}{\partial G_2} = bCG_1^a G_2^{b-1} - \lambda \beta P_2 = 0 \rightarrow \frac{bU(G_1, G_2)}{G_2} = \lambda \beta P_2$$

$$\frac{\partial L}{\partial \lambda} = TM - A - \alpha G_1 P_1 - \beta G_2 P_2 = 0 \text{ (Budget Constraint)}$$

Marginal Rate of Substitution: The MRS represents the rate at which the consumer is willing to trade G_2 for G_1 while maintaining the same level of utility. For the Cobb-Douglas utility function:

$$MRS = {MU_{G_1}}/{MU_{G_2}} = \frac{\partial U / \partial G_1}{\partial U / \partial G_2} = \frac{a}{b}\frac{G_2}{G_1}$$

4.3.2. Second-Order Conditions (SOCs)

The bordered Hessian matrix must be negative definite to ensure a maximum. The bordered Hessian is:

$$\begin{bmatrix} 0 & -\alpha P_1 & -\beta P_2 \\ -\alpha P_1 & \frac{a(a-1)U}{G_1^2} & \frac{abU}{G_1 G_2} \\ -\beta P_2 & \frac{abU}{G_1 G_2} & \frac{b(b-1)U}{G_2^2} \end{bmatrix}$$

The determinant of the bordered Hessian (D) is

$$D = \frac{2\alpha\beta P_1 P_2 abU}{G_1 G_2} - \frac{a(a-1)(\beta P_2)^2 U}{G_1^2} - \frac{b(b-1)(\alpha P_1)^2 U}{G_2^2}$$

Since $0 < a < 1$ and $0 < b < 1$, and U is positive, $D > 0$. This satisfies the second-order condition for a maximum.

4.4. Solving for Optimal Demands

From FOCs 1 and 2: If α is positive: $\frac{aU}{\alpha P_1 G_1} = \lambda$ If β is positive: $\frac{bU}{\beta P_2 G_2} = \lambda$ If α , and β are negative the same equations are valid.

Equating and rearranging:

$$G_2 = \frac{b\alpha P_1 G_1}{a\beta P_2}$$

Substitute this expression for G2 into the budget constraint (FOC 3):

$$TM - A - \alpha G_1 P_1 - \beta \frac{b\alpha P_1 G_1}{a\beta P_2} P_2 = 0$$

Simplifying the equation:

$TM - A = \alpha G_1 P_1 \frac{a+b}{a}$ Solving for G_1^* and G_2^*:

$$G_1^* = \frac{a}{a+b} \frac{TM - A}{\alpha P_1}$$

$$G_2^* = \frac{b}{a+b} \frac{TM - A}{\beta P_2}$$

5. Results and Discussion

The solutions for G_1^* and G_2^* represent the optimal quantities of goods 1 and 2 that the consumer will choose, given their preferences, prices, total money, fixed budget component, and their decision to be a buyer or seller of each good (as reflected in α and β).

5.1. The Optimality Condition: MRS and Effective Price Ratio

The most important result is derived by combining the first-order conditions:

$$\frac{\frac{aU}{G_1}}{\frac{bU}{G_2}} = \frac{\lambda\alpha P_1}{\lambda\beta P_2}$$

Simplifying, and using the definition of the MRS:

$$MRS = \frac{a}{b}\frac{G_2}{G_1} = \frac{\alpha P_1}{\beta P_2}$$

This equation states that at the optimum, the consumer's MRS *must* equal the *effective* price ratio. The effective price ratio is not simply P_1/P_2; it's $\alpha P_1/\beta P_2$. The α and β terms are crucial: they transform the market prices into *effective* prices that reflect the consumer's trading position. If the consumer is a buyer of a good, the corresponding α or β is 1, and the effective price is the market price. If the consumer is a *seller* of a good, the corresponding α or β is -1, and the effective price is the *negative* of the market price, representing the income received from selling the good.

5.2. Simultaneous Substitution as the Driving Force

If the MRS is *not* equal to the effective price ratio, the consumer is *not* at an optimum. They can *increase* their utility by engaging in simultaneous substitution:

If $MRS > \alpha P_1/\beta P_2$: The consumer values G_1 relatively more than the market (as reflected in the effective price ratio). They will *sell* some G_2 (setting $\beta = -1$) and use the proceeds to *buy* more G_1 (setting $\alpha = 1$). This moves them along the budget line to a higher indifference curve.

If $MRS < \alpha P_1/\beta P_2$: The consumer values G_2 relatively more. They will *sell* some G_1 (setting $\alpha = -1$) and use the proceeds to *buy* more G_2 (setting $\beta = 1$). This also moves them along the budget line to a higher indifference curve. The model, therefore, highlights that consumers are motivated to adjust both quantities *simultaneously* to exploit differences between their internal valuation (MRS) and the market opportunities (effective prices).

5.3. The Role of 'A'

The fixed budget component, A, shifts the budget constraint. A positive A expands the feasible consumption set, allowing for higher utility. A negative A contracts the set. While A affects the *level* of utility attainable, it does *not* change the fundamental optimality condition: $MRS = \alpha P_1/\beta P_2$.

5.4. Expenditure Shares

When $\alpha = \beta = 1$ (the consumer buys both goods), the model exhibits the standard Cobb-Douglas property of constant expenditure shares. The consumer spends a fraction $a/a+b$ of their "available income" $TM - A$ on G_1 and $b/a+b$ on G_2. However, with endogenous trading (α and/or $\beta = -1$), expenditure shares are no longer constant and become dependent on all parameters of the model.

This model offers a simultaneous substitution of utility maximization. It can be summarized as SUM model.

5.5. Extendibility of SUM Model to the EKB Model

This paper is fully applicable to the Engel–Kollat–Blackwell (EKB) model in contemporary consumer behavior. It can be regarded as an extension of the EKB model by incorporating simultaneous trading and utility maximization in contemporary consumer behavior.

The EKB model is a widely used framework for understanding consumer decision-making. Recent studies have applied the model to various contexts, including simultaneous buying and selling behaviors. This extension enriches the theoretical foundation of the EKB model by highlighting its adaptability to dynamic market conditions.

6. Application and Implementation of the Model in Real-World Scenarios

The theoretical model of small-scale trade and consumer behavior can be observed in practice through several real-world examples. These cases illustrate how, despite political tensions, geographical barriers, or technological transformations, local people continue to exchange goods and services in ways that sustain livelihoods, preserve cultural traditions, and adapt to modern challenges.

6.1. Pakistan–India Trade

Despite the persistent tensions and political disputes between Pakistan and India, local traders have maintained limited cross-border trade. At the Wagah–Attari border, communities exchange goods such as spices, dry fruits, and textiles, which are central to both cultural consumption and everyday needs (Lalwani & Byrne, 2013). In Kashmir, the exchange is more artisanal, with handmade shawls and other handicrafts crossing the Line of Control (LoC).

These exchanges demonstrate how local economic interactions can continue even under strained political conditions, providing resilience and sustaining community ties (Tremblay & Bhatia, 2018).

6.2. Turkey–Azerbaijan Trade

Trade between Turkey and Azerbaijan, particularly between Nakhchivan (Azerbaijan) and Iğdır (Turkey), reflects strong historical, cultural, and linguistic ties. Local traders frequently exchange goods such as fruits, vegetables, and livestock across the border, ensuring food security and economic interaction (Huseynov & Bayraktar, 2019). Beyond basic necessities, cultural products such as handmade carpets and rugs are also exchanged, representing not just economic activity but the preservation of shared cultural heritage (Tabrizi & Gürsan-Salzmann, 2017).

6.3. Mexico–Southern Neighbors Trade

In Mexico's southern border region, economic exchanges remain an important livelihood strategy for local traders. Goods such as coffee, handicrafts, and textiles are exchanged with Guatemalan and Belizean counterparts (Morales, 2017). These trades often extend to fresh produce and livestock, which Mexican traders purchase from their Central American neighbors (Rosenberg, 2018). Such activities not only sustain household economies but also promote cross-cultural interaction, reflecting how border trade can play a stabilizing role in regions with shared needs and traditions.

6.4. Contemporary Consumer Behavior in Digital Trading

In the digital economy, trade and consumer behavior have undergone rapid transformation. A McKinsey report (June 2025) shows that consumers are spending significantly more time online, with digital platforms becoming central to purchase decisions. Social media, online marketplaces, and e-commerce platforms are influencing consumer preferences by shaping perceptions of value and brand recognition. The importance of local brands is being redefined as they compete and coexist with global firms in the digital space. An implementation of this model highlights the growing significance of digital transactions in shaping not only consumer behavior but also cultural and lifestyle patterns. In the digital era, transactions are no longer limited to physical exchanges; they involve new forms of trust, branding, and value perception. Consequently, consumer lifestyles are evolving rapidly, reflecting a shift toward convenience, personalization, and global interconnectedness.

As it can be observed, this paper by introducing the SUM model shows that it is applicable and may be implemented in numerous cases and examples in the real world.

7. Conclusion

This paper presents a model of consumer choice that departs from the standard framework by explicitly incorporating *endogenous trading* and *simultaneous substitution*. Consumers are not simply passive buyers; they actively choose whether to buy or sell each good, adjusting their consumption bundle to maximize utility. The model demonstrates that the driving force behind this substitution is the discrepancy between the consumer's Marginal Rate of Substitution (MRS) and the *effective* price ratio, which incorporates the buy/sell decision for each good. The fixed budget component, A, further enriches the model by allowing for savings, borrowing, or pre-committed expenditures.

This framework of this SUM model offers a more realistic and nuanced perspective on consumer behavior, applicable to situations involving resale markets, barter, intra-household exchange, and any context where individuals can actively trade goods to improve their well-being. The model provides a solid foundation for future research, including empirical testing of its predictions, extensions to dynamic settings to more fully capture intertemporal choice, and the incorporation of heterogeneous consumer preferences and trading costs. By explicitly modeling the *process* of utility-enhancing trade, this paper contributes to a deeper understanding of the micro-foundations of market interactions.

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